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Determination and Verification of Thermal Response Factors for Thermal Conduction Applications

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ABSTRACT

New formulas for calculating thermal response factors for multiple-layer construction have been developed by a rigorous derivation. A comparison was made of the time for computation between the presently used matrix algebra method and the method given in this paper. Results were obtained using the new method in one-fiftieth to one-half of the computational time necessary to obtain solutions from the matrix algebra method.

Comparisons with another analytical method were performed to verify the accuracy of the response-factor technique.

Key Words: Dynamic conduction heat transfer; heat transfer; thermal response factor; verification.

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1. INTRODUCTION

The analytical solutions needed to describe temperature or heat flow for steady periodic or transient conduction heat transfer in multi-layer walls, ceilings and floors are quite complicated and are not available for non-linear conditions such as thermal radiation at surfaces and time-dependent changes in surface film coefficients. It has therefore been expedient to employ "approximation" methods by which non-linear conditions may be satisfied. One such method, which is the result of an analytical formulation, is termed the "response-factor method," and solves for temperatures or heat flows of multi-layer constructions based on the past temperature history.

A response-factor method defined by Kusuda [1]* uses overlapping triangular pulses to compute response factors for a particular construction. These response factors are then used to determine temperatures and heat flows in response to previously occurring events. In order to handle multi-layer constructions for solution of response factors by computers, a matrix algebra was developed for an arbitrary number of layers. There are some inherent difficulties in using this approach, in that the calculation of response factors can be quite time consuming because certain functions

* See references cited at end of text

are not well suited for efficient calculation. A portion of this paper will be devoted to the development of equations which allow for increased computational efficiency. The algorithms presented are sufficient for the determination of up to a seven-layer composite, which is probably more than sufficient for possible building constructions.

Unfortunately, the accuracy of the temperatures and heat flows values calculated by response-factor methods has received very little verification. Although it is well known that the thermal properties of building materials are not as well defined as they should be, this is no excuse for not knowing the error incurred by the use of an approximate method for whatever thermal properties may be assigned to a particular program for solution. A portion of this paper will be devoted to a comparison between numerical results from an analytical solution and results from calculations using the response-factor technique.

2. ANALYSIS

For one-dimensional heat flow in an individual layer of one or more parallel layers, the partial differential equation is given by

$$\frac{\partial^2 \nu_m}{\partial x^2} = \frac{1}{\alpha_m} \frac{\partial \nu_m}{\partial t} \quad . \tag{1}$$

where v_m is the temperature potential above a datum plane, x is a dimension along which heat is flowing, α_m is the thermal diffusivity of the layer material, and t is the time. For continuity of temperature and heat flow

between layers, perfect contact is assumed, i.e.,

$$\nu_{m-1} = \nu_m$$

and

$$K_{m-1} \frac{d\nu_{m-1}}{dx} = K_m \frac{d\nu_m}{dx} \tag{2}$$

where K_m is the thermal conductivity of the respective layer. Applying the Laplace transform to (1) gives

$$\frac{d^2\bar{\nu}_m}{dx^2} = q_m^2 \bar{\nu} , \quad q_m^2 = \frac{p}{\alpha_m} \tag{3}$$

for which a solution is:

$$\bar{\nu}_m = A_m e^{q_m(x-b_{m-1})} + B_m e^{-q_m(x-b_{m-1})}$$

$$K_m \frac{d\bar{\nu}_m}{dx} = K_m q_m \left[A_m e^{q_m(x-b_{m-1})} - B_m e^{-q_m(x-b_{m-1})} \right]$$

where b_{m-1} is the distance from x=0 (surface of first layer) to the nearest face of the m-th layer. Applying the continuity conditions (2) gives the following relations

$$\frac{2 A_{m-1}}{(1 + \sigma_{m-1})} = A_m e^{-y_{m-1}} + k_{m-1} B_m e^{-y_{m-1}}$$

$$\frac{2 B_{m-1}}{(1 + \sigma_{m-1})} = k_{m-1} A_m e^{y_{m-1}} + B_m e^{y_{m-1}}$$

where

$$\sigma_{m-1} = \frac{K_m}{K_{m-1}} \sqrt{\frac{\alpha_{m-1}}{\alpha_m}}$$

$$k_{m-1} = \frac{1 - \sigma_{m-1}}{1 + \sigma_{m-1}}$$

$$y_{m-1} = \frac{(b_{m-1} - b_{m-2}) \sqrt{p}}{\sqrt{\alpha_{m-1}}} = \frac{\ell_{m-1} \sqrt{p}}{\sqrt{\alpha_{m-1}}}$$

and ℓ_{m-1} is the thickness of the (m-1)-th layer. This process is continued until the constants A_1 and B_1 (pertaining to the layer with face exposed at x=0) are found in terms of A_n and B_n (pertaining to the n-th layer with face exposed at $x=b_n$). At x=0, the heat flux is proportional to the temperature difference between the fluid (air, gas or liquid) and the surface, and is represented by the relationship

$$-R_1 K_1 \frac{d\nu_1}{dx} = f(t) - \nu_1 \tag{4}$$

where R_1 is the surface film resistance and f(t) is fluid temperature as a function of time. Similarly, the boundary condition at $x=b_n$ is

$$-R_2 K_n \frac{d\nu_n}{dx} = \nu_n - g(t) \tag{5}$$

where R_2 is the surface film resistance and g(t) is a fluid temperature as a function of time. When either R_1 or R_2 is zero, the time temperature function represents temperatures at the respective surfaces. The resulting expressions for the transform of the temperatures in layer 1 and layer n are given by

$$\bar{v}_1 = \frac{\bar{f}(p)}{W}\left[P_1+Q_1+V_2\sqrt{p}\,(S_1+T_1)\right] + \frac{H\bar{g}(p)}{W}\left[\sinh x\sqrt{\frac{p}{\alpha_1}} + V_1\sqrt{p}\cosh x\sqrt{\frac{p}{\alpha_1}}\right] \quad (6)$$

and

$$\bar{v}_n = \frac{G\bar{f}(p)}{W}\left[\sinh\sqrt{\frac{p}{\alpha_n}}\,(b_n-x) + V_2\sqrt{p}\cosh\sqrt{\frac{p}{\alpha_n}}\,(b_n-x)\right] +$$

$$+ \frac{\bar{g}(p)}{W}\left[P_2+Q_2+V_1\sqrt{p}\,(S_2-T_2)\right] \quad (7)$$

where

$P_1 = \Sigma\, J_m \sinh\sqrt{p}\,(N_m-x/\sqrt{\alpha_1})$ $\quad Q_1 = \Sigma\, L_m \sinh\sqrt{p}\,(E_m+x/\sqrt{\alpha_1})$

$S_1 = \Sigma\, J_m \cosh\sqrt{p}\,(N_m-x/\sqrt{\alpha_1})$ $\quad T_1 = \Sigma\, L_m \cosh\sqrt{p}\,(E_m+x/\sqrt{\alpha_1})$

$P_2 = \Sigma\, J_m \sinh\sqrt{p}\,(N_m-(b_n-x)/\sqrt{\alpha_n})$ $\quad Q_2 = \Sigma\, L_m \sinh\sqrt{p}\,(E_m-(b_n-x)/\sqrt{\alpha_n})$

$S_2 = \Sigma\, J_m \cosh\sqrt{p}\,(N_m-(b_n-x)/\sqrt{\alpha_n})$ $\quad T_2 = \Sigma\, L_m \cosh\sqrt{p}\,(E_m-(b_n-x)/\sqrt{\alpha_n})$

$P = \Sigma\, J_m \sinh\sqrt{p}\, N_m$ $\quad Q = \Sigma\, L_m \sinh\sqrt{p}\, E_m$

$S = \Sigma\, J_m \cosh\sqrt{p}\, N_m$ $\quad T = \Sigma\, L_m \cosh\sqrt{p}\, E_m$

(The above summations are over $m=1,2,\ldots\ 2^{n-2}$, n being the number of layers.)

$$W = P + Q + V_1V_2p(P-Q) + \sqrt{p}\left[V_2(S+T) + V_1(S-T)\right]$$

$$G = 2^{n-1}/(1+\alpha_1)(1+\alpha_2)\cdots(1+\sigma_{n-1}), \quad H = \frac{GK_n}{K_1}\sqrt{\frac{\alpha_1}{\alpha_n}}$$

$$N_m = \sum_{i=1}^{n} A_{i,m}\frac{\ell_i}{\sqrt{\alpha_i}}, \quad E_m = N_m - \frac{2\ell_1}{\sqrt{\alpha_1}}, \quad A_{1,m} = 1$$

$$V_1 = R_1K_1/\sqrt{\alpha_1}, \quad V_2 = R_2K_n/\sqrt{\alpha_n}$$

and J_m, L_m and $A_{i,m}$ are defined in Table 1.

TABLE 1. Definition For J_m, L_m and $A_{i,m}$

m	J_m	L_m	$A_{2,m}$	$A_{3,m}$	$A_{4,m}$	$A_{5,m}$	$A_{6,m}$	$A_{7,m}$
1	1	k_1	1	1	1	1	1	1
2	k_1k_2	k_2	-1	1	1	1	1	1
3	k_1k_3	k_3	-1	-1	1	1	1	1
4	k_2k_3	$k_1k_2k_3$	1	-1	-1	1	1	1
5	k_1k_4	k_4	-1	-1	-1	1	1	1
6	k_2k_4	$k_1k_2k_4$	1	-1	-1	1	1	1
7	k_3k_4	$k_1k_3k_4$	1	1	-1	1	1	1
8	$k_1k_2k_3k_4$	$k_2k_3k_4$	-1	1	-1	1	1	1
9	k_4k_5	$k_1k_4k_5$	1	1	1	-1	1	1
10	k_3k_5	$k_1k_3k_5$	1	1	-1	-1	1	1
11	k_2k_5	$k_1k_2k_5$	1	-1	-1	-1	1	1
12	k_1k_5	k_5	-1	-1	-1	-1	1	1
13	$k_1k_2k_3k_5$	$k_2k_3k_5$	-1	1	-1	-1	1	1
14	$k_1k_2k_4k_5$	$k_2k_4k_5$	-1	1	1	-1	1	1
15	$k_1k_3k_4k_5$	$k_3k_4k_5$	-1	-1	1	-1	1	1
16	$k_2k_3k_4k_5$	$k_1k_2k_3k_4k_5$	1	-1	1	-1	1	1
17	k_5k_6	$k_1k_5k_6$	1	1	1	1	-1	1
18	k_4k_6	$k_1k_4k_6$	1	1	1	-1	-1	1
19	k_3k_6	$k_1k_3k_6$	1	1	-1	-1	-1	1
20	k_2k_6	$k_1k_2k_6$	1	-1	-1	-1	-1	1
21	k_1k_6	k_6	-1	-1	-1	-1	-1	1
22	$k_1k_2k_3k_6$	$k_2k_3k_6$	-1	1	-1	-1	-1	1
23	$k_1k_2k_4k_6$	$k_2k_4k_6$	-1	1	1	-1	-1	1
24	$k_1k_2k_5k_6$	$k_2k_5k_6$	-1	1	1	1	-1	1
25	$k_1k_3k_4k_6$	$k_3k_4k_6$	-1	-1	1	-1	-1	1
26	$k_1k_3k_5k_6$	$k_3k_5k_6$	-1	-1	1	1	-1	1
27	$k_1k_4k_5k_6$	$k_4k_5k_6$	-1	-1	-1	1	-1	1
28	$k_2k_3k_4k_6$	$k_1k_2k_3k_4k_6$	1	-1	1	-1	-1	1
29	$k_2k_3k_5k_6$	$k_1k_2k_3k_5k_6$	1	-1	1	1	-1	1
30	$k_2k_4k_5k_6$	$k_1k_2k_4k_5k_6$	1	-1	-1	1	-1	1
31	$k_3k_4k_5k_6$	$k_1k_3k_4k_5k_6$	1	1	-1	1	-1	1
32	$k_1k_2k_3k_4k_5k_6$	$k_2k_3k_4k_5k_6$	-1	1	-1	1	-1	1

The transforms of the heat flux at x=0 and at $x=b_n$ are found by differentiating (6) and (7) with respect to x and multiplying by minus one and the respective thermal conductivity:

$$\bar{F}_1 = \frac{K_1\sqrt{p}}{W\sqrt{\alpha_n}}\left[\left\{S-T + V_2\sqrt{p}\ (P-Q)\right\}\bar{f}(p) - H\bar{g}(p)\right] \tag{8}$$

$$\bar{F}_n = \frac{K_n\sqrt{p}}{W\sqrt{\alpha_n}}\left[G\bar{f}(p) - \left\{S+T + V_1\sqrt{p}\ (P-Q)\right\}\bar{g}(p)\right] . \tag{9}$$

The inversion of (8) and (9) is performed by evaluating the residues at the poles of the denominator W=0, where $p= -\beta^2$ or $\sqrt{p} = i\beta$, which gives the relationship

$$W_\beta = (1-V_1V_2\beta^2)\ \Sigma J_m \sin N_m\beta + (1+V_1V_2\beta^2)\ \Sigma L_m \sin E_m\beta$$

$$+ \beta\left[(V_2+V_1)\ \Sigma J_m \cos N_m\beta + (V_2-V_1)\ \Sigma L_m \cos E_m\beta\right] = 0 \tag{10}$$

and the differentiation of W with respect to p evaluated at $p= -\beta^2$ gives

$$U = (1-V_1V_2\beta^2)\ \Sigma J_m N_m \cos N_m\beta + (1+V_1V_2\beta^2)\ \Sigma L_m E_m \cos E_m\beta$$

$$+ (V_2+V_1)\left[\Sigma J_m \cos N_m\beta - \beta\ \Sigma J_m N_m \sin N_m\beta\right]$$

$$+ (V_2-V_1)\left[\Sigma L_m \cos E_m\beta - \beta\ \Sigma L_m E_m \sin E_m\beta\right]$$

$$- 2\beta V_1V_2\left[\Sigma J_m \sin N_m\beta - \Sigma L_m \sin E_m\beta\right] \tag{11}$$

or $\quad 2i\beta\left(\frac{dW}{dp}\right)_{p=-\beta^2} = U .$

The residues at the poles $p = -\beta^2$ are

$$F_{1\beta} = -\frac{2K_1}{\sqrt{\alpha_1}} \sum \frac{\beta_i^2}{U_i} \left[\bar{f}\,(-\beta_i^2)(D_1 - V_2 D_2 \beta) - H\bar{g}(-\beta_i^2) \right] e^{-\beta_i^2 t} \tag{12}$$

and

$$F_{n\beta} = -\frac{2K_n}{\sqrt{\alpha_n}} \sum \frac{\beta_i^2}{U_i} \left[G\bar{f}(-\beta_i^2) - (D_3 - V_1 D_2 \beta)\, \bar{g}(-\beta_i^2) \right] e^{-\beta_i^2 t} \tag{13}$$

where

$$D_1 = \Sigma(J_m \cos N_m\beta - L_m \cos E_m\beta), \quad D_2 = \Sigma\,(J_m \cos N_m\beta + L_m \cos E_m\beta),$$

$$D_3 = \Sigma(J_m \sin N_m\beta - L_m \sin E_m\beta).$$

Of particular concern is the evaluation of the first root of (10). This can be done expeditiously by expanding the sines and cosines in their series and considering only the first two terms, in order to obtain an initial estimate of the first root

$$W_\beta \simeq A_1\beta - A_2\beta^3 = 0$$

or

$$\beta_1^2 \approx A_1/A_2 \tag{14}$$

with

$$A_1 = \Sigma\,(J_m N_m + L_m E_m) + (V_1 + V_2)\,\Sigma\, J_m + (V_2 - V_1)\,\Sigma\, L_m$$

and

$$A_2 = V_1 V_2\ \Sigma\,(J_m N_m - L_m E_m) + \frac{V_1 + V_2}{2}\,\Sigma\, J_m N_m^2 + \frac{V_2 - V_1}{2}\,\Sigma\, L_m E_m^2$$

$$+ \frac{1}{6}\,\Sigma\,(J_m N_m^3 + L_m E_m^3)\quad .$$

Consider the triangular pulse function of Kusuda [1], where

$$
\begin{array}{llll}
f(t) = 0 & \bar{f}(p) = 0 & & t \le 0 \\
\quad = t/\delta & \quad = 1/\delta p^2 & & 0 < t \le \delta \\
\quad = 2-t/\delta & \quad = (1-2e^{-p\delta})/\delta p^2 & & \delta < t \le 2\delta \\
\quad = 0 & \quad = (1-e^{-p\delta})^2/\delta p^2 & & t > 2\delta
\end{array}
$$

which when substituted for $\bar{f}(p)$ and $\bar{g}(p)$ in (8) and (9) gives double poles at $p=0$. Following are limits of the necessary functions of (8) and (9) for evaluating the residues at the double poles.

$$\lim_{p \to o} \left(\frac{W}{\sqrt{p}} \right) = A_1 + A_2 p$$

$$\lim_{p \to o} \quad S - T + V_2 \sqrt{p}\ (P-Q) = B_1 + B_2 p$$

$$\lim_{p \to o} \quad S + T + V_1 \sqrt{p}\ (P=Q) = C_1 + C_2 p$$

A_1 and A_2 are defined by (14) and

$$B_1 = \Sigma\ (J_m - L_m), \qquad C_1 = \Sigma\ (J_m + L_m)$$

$$B_2 = \frac{1}{2}\ \Sigma\ (J_m N_m^{\ 2} - L_m E_m^{\ 2}) + V_2\ \Sigma\ (J_m N_m - L_m E_m)$$

$$C_2 = \frac{1}{2}\ \Sigma\ (J_m N_m^{\ 2} + L_m E_m^{\ 2}) + V_1\ \Sigma\ (J_m N_m - L_m E_m).$$

For the first term in (8), the residue for $0<t\le\delta$ is

$$\bar{X}_t = \frac{K_1}{\delta\sqrt{\alpha_1}} \left[\frac{tB_1}{A_1} + \frac{A_1 B_2 - A_2 B_1}{A_1^{\ 2}} \right] \tag{16}$$

for the last term in (9), the residue is

$$\bar{Z}_t = \frac{K_n}{\delta\sqrt{\alpha_n}}\left[\frac{tC_1}{A_1} + \frac{A_1C_2-A_2C_1}{A_1^2}\right] \tag{17}$$

and for the first term in (9) and the last term in (8), the residue is

$$\bar{Y}_t = \frac{K_nG}{\delta\sqrt{\alpha_n}}\left[\frac{t}{A_1} - \frac{A_2}{A_1^2}\right] \tag{18}$$

where

$$\frac{K_1B_1}{A_1\sqrt{\alpha_1}} = \frac{K_nC_1}{A_1\sqrt{\alpha_n}} = \frac{K_nG}{A_1\sqrt{\alpha_n}} = \frac{1}{R}$$

and R is the total thermal resistance of the n-layers plus R_1 and R_2.

The response factors X_1, Y_1 and Z_1 evaluated at $t=\delta$, become

$$X_1 = \bar{X}_\delta - \frac{K_1}{\sqrt{\alpha_1}}\sum(D_1 - V_2D_2\beta_i)\,\psi_i$$

$$Y_1 = \bar{Y}_\delta - \frac{K_nG}{\sqrt{\alpha_n}}\sum\psi_i \tag{19}$$

$$Z_1 = \bar{Z}_\delta - \frac{K_n}{\sqrt{\alpha_n}}\sum(D_3 - V_1D_2\beta_i)\,\psi_i$$

where

$$\psi_i = \frac{2}{\delta}\;\frac{e^{-\beta_i^2}}{\beta_i^2\,U_i}\;.$$

For X_2, Y_2 and Z_2 evaluated at $t=2\delta$,

$$X_2 = \frac{1}{R} - \bar{X}_\delta - \frac{K_1}{\sqrt{\alpha_1}} \sum (D_1 - V_2 D_2 \beta_i) \, \psi_i (e^{-\beta_i^2 \delta} - 2)$$

$$Y_2 = \frac{1}{R} - \bar{Y}_\delta - \frac{K_n G}{\sqrt{\alpha_n}} \sum \psi_i (e^{-\beta_i^2} - 2) \qquad (20)$$

$$Z_2 = \frac{1}{R} - \bar{Z}_\delta - \frac{K_n}{\sqrt{\alpha_n}} \sum (D_3 - V_1 D_2 \beta_i) \, \psi_i (e^{-\beta_i^2 \delta} - 2) \quad ,$$

and for $t>2\delta$,

$$X_j = - \frac{K_1}{\sqrt{\alpha_1}} \sum (D_1 - V_2 D_2 \beta_i) \, \psi_i \left(1 - e^{-\beta_i^2 \delta}\right)^2 e^{-(j-3)\beta_i^2 \delta} \qquad (21)$$

For larger values of j, the response factors decrease with increase in j by a common ratio — i.e., for N sufficiently large,

$$\overline{CR} = e^{-\beta_1^2 \delta} = \frac{X_{j+1}}{X_j} = \frac{Y_{j+1}}{Y_j} = \frac{Z_{j+1}}{Z_j} \qquad (22)$$

for all $j \geq N$, where β_1 is the first root of (10). Thus for j greater than N, the response factors can be computed by the relationship

$$X_{j+1} = X_j \cdot \overline{CR} \quad .$$

The temperatures f(t) and g(t) may be constructed from a series of triangular pulse functions (15), overlapping in time by an amount δ. By the principle of superposition, the heat flux at time τ is determined from the sum of the products of the response factors and the temperatures from the present time to preceding time intervals of duration δ. The heat flow at time $t=\tau$ and

at the x=0 face is then

$$F_{1,\tau} = \sum_{i=1} (X_i V_{1,\tau-i+1} - Y_i V_{n,\tau-i+1}) \tag{23}$$

and the heat flux at $x=b_n$ is

$$F_{n,\tau} = \sum_{i=1} (Y_i V_{1,\tau-i+1} - Z_i V_{n,\tau-i+1}) \tag{24}$$

where V is the temperature potential in relation to a fixed datum plane temperature, which is referenced to time, $t=\tau$, $\tau-1$, $\tau-2$, etc.

For certain types of constructions, the number of required response factors can become quite large to give a reasonable accuracy to the heat fluxes of (23) and (24). Conduction transfer functions can reduce this number considerably and are defined from the relationship, $F_{1,\tau} - CR\ F_{1,\tau-1}$ of (23) and similarly from (24). The conduction transfer functions are then defined as follows:

$$X_1' = X_1 \qquad Y_1' = Y_1 \qquad Z_1' = Z_1$$

$$X_j' = X_j - \overline{CR}\ X_{j-1} \qquad Y_j' = Y_j - \overline{CR}\ Y_{j-1} \qquad Z_j' = Z_j - \overline{CR}\ Z_{j-1} \tag{25}$$

Using these functions, the heat flux at $t=\tau$ and $x=0$ becomes

$$F_{1,\tau} = \overline{CR} \cdot F_{1,\tau-1} + \sum_{j=1} (X_j' V_{1,\tau-j+1} - Y_j' V_{n,\tau-j+1})$$

From (21) and (25), it can be seen that the conduction transfer functions are numerically very small numbers as j approaches N, and the number of functions needed for computation purposes is considerably reduced. However,

initially it is necessary to know the value of $F_{1,\tau-1}$. This must be determined from several iterations over the past temperature history of V_1 and V_n.

3. RESPONSE FACTORS

Using the algorithms of the previous section, the response factors of equations (19), (20), and (21) are calculated from the computer program found in Appendix A. To show time savings in response-factor calculations, identical multi-layer constructions were input into both the program of Kusuda [1] and the program of Appendix A. Six multi-layer constructions are shown in Table 2, and the computation times for the two programs are shown in Table 3. As can be seen, considerable time saving is available from the program of Appendix A.

In the example of Table 2, there is an evident lack of enclosed air spaces in the building construction. Kusuda [2] assumed only a purely thermal resistance effect of air spaces during dynamic temperature changes. It is the supposition of the author that both the air space thickness and the heat capacity of the air should be considered for heat transfer calculations. From literature values, the thermal diffusivity of dry air varies from 0.639 ft^2/h at 0 °F to .977 ft^2/h at 120 °F, and these values are reduced somewhat by the presence of water vapor. For the program of Appendix A, the thermal diffusivity of air in air spaces is assumed to be 0.75 ft^2/h, the air space thickness is assumed to be one inch if not specified by the input card, and the thermal resistance is as defined for steady-state air-space values.

Table 2. Multi-Layer Constructions

	Layer Description	Layer Thickness ft	Thermal Conductivity Btu/h ft F	Density lb/ft^3	Specific Heat Btu/lb F
Case 1 Two Layers					
1	1-IN MINERAL FIBERBOARD	0.833	.035	23.0	.140
2	4-IN LIGHT-WEIGHT CONCRETE	.3333	.100	40.0	.200
Case 2 Three Layers					
1	5/8-IN PLASTER BOARD	.0521	.094	50.0	.260
2	3-1/2 IN BATT INSULATION	.2917	.026	2.0	.220
3	3/4-IN WOOD SIDING	.0625	.800	36.0	.280
Case 3 Four Layers					
1	1/2-IN PLASTER BOARD	.0417	.094	50.0	.260
2	3/4-IN POLYSTYRENE INSULATION	.0625	.094	50.0	.260
3	4-IN COMMON BRICK	.3333	.417	120.0	.190
4	4-IN FACE BRICK	.3333	.750	130.0	.190
Case 4 Five Layers					
1	1/2-IN PLASTER BOARD	.0417	.094	50.0	.260
2	1-IN BATT INSULATION	.0833	.026	2.0	.220
3	4-IN L.W. CONCRETE	.3333	.100	40.0	.200
4	3/4-IN BOARD INSULATION	.0625	.017	2.2	.290
5	1-IN STUCCO	.0833	.400	116.0	.200
Case 5 Six Layers					
1	3/4-IN ACOUSTIC TILE	.0625	.035	30.0	.200
2	4-IN HEAVY WEIGHT CONCRETE	.3333	1.000	140.0	.200
3	2-IN ROOF INSULATION	.1667	.025	5.7	.200
4	2-IN H.W. CONCRETE	.1667	1.000	140.0	.200
5	3/8-IN FELT	.0313	.110	70.0	.400
6	1/2-IN SLAG	.0417	.830	55.0	.400
Case 6 Seven Layers					
1	1/2-IN PLASTER BOARD	.0417	.094	50.0	.260
2	1-IN BATT INSULATION	.0833	.026	2.0	.220
3	1/2-IN PLYWOOD	.0417	.067	34.0	.290
4	3-IN H.W. CONCRETE	.2500	1.000	140.0	.200
5	1/2-IN PLYWOOD	.0417	.067	34.0	.290
6	3/4-IN BOARD INSULATION	.0625	.017	2.2	.290
7	1-IN STUCCO	.0833	.400	116.0	.200

Table 3. Execution Time for Computation of Response Factors, Seconds

	Kusuda [1]	Appendix A	Time Savings
Two layers	2.037	.187	1.850
Three layers	8.885	.174	8.711
Four layers	1.902	.378	1.524
Five layers	3.018	.625	2.393
Six layers	1.609	.846	.763
Seven layers	9.786	1.681	8.105

Table 4. Heat Flux at Inside Surface of Wood Frame Construction with Air Space, Btu/h ft^2

Time (hr)	No heat capacity	3 1/2" air space	5 1/2" air space
1	1.742	1.021	.954
2	1.262	1.242	1.203
3	1.359	1.342	1.319
4	1.548	1.525	1.494
5	1.707	1.691	1.666
6	.964	1.109	1.233
7	- 1.966	- 1.598	- 1.154
8	- 6.113	- 5.684	- 5.024
9	-10.410	- 9.924	- 9.214
10	-14.248	-13.809	-13.144
11	-17.427	-17.065	-16.501
12	-19.720	-19.463	-19.042
13	-20.908	-20.783	-20.544
14	-20.827	-20.849	-20.822
15	-19.497	-19.660	-19.846
16	-17.038	-17.334	-17.718
17	-13.592	-13.996	-14.552
18	- 9.416	- 9.908	-10.599
19	- 4.960	- 5.459	- 6.201
20	- 1.785	- 2.088	- 2.639
21	- .746	- .870	- 1.128
22	- .214	- .281	- .414
23	+ .239	+ .185	+ .093
24	.680	.628	.547

The introduction of these changes does alter the response factors and heat flux amplitudes in response to external temperature variations. Table 4 gives heat flux values at the inside surface for a wood-frame construction, with no heat capacity [2], and 3 1/2 and 5 1/2 inch air spaces. The external temperature variation is taken from Figure 1.

4. VERIFICATION OF RESPONSE FACTORS

Response factors as defined in this paper are the result of an analytical formulation which employs a past time history of overlapping pulses for the temperatures at or adjacent to surfaces of composite solids to determine the heat flux or temperature at present time. A linear variation in temperature over the time period, (usually one hour) is assumed. The ability of response factors to give accurate values for heat flux has been questioned. For this reason, it is appropriate to compare the numerical results from the response-factor calculation with those from another analytical calculation.

Analytical solutions can be found from equations (6) and (7), where the temperature-time functions can be defined by the trigonometric series,

$$f(t) = \Sigma\,(A_n \cos W_n t + B_n \sin W_n t)\ ; \qquad \bar{f}(p) = \Sigma\,\frac{A_n P + B_n W_n}{p^2 + W_n^2} \qquad (27)$$

and

$$g(t) = C_o + \Sigma\,(C_n \cos W_n t + D_n \sin W_n t)\ ; \quad \bar{g}(p) = \frac{C_o}{p} + \Sigma\,\frac{C_n p + D_n W_n}{p^2 + W_n^2} \qquad (28)$$

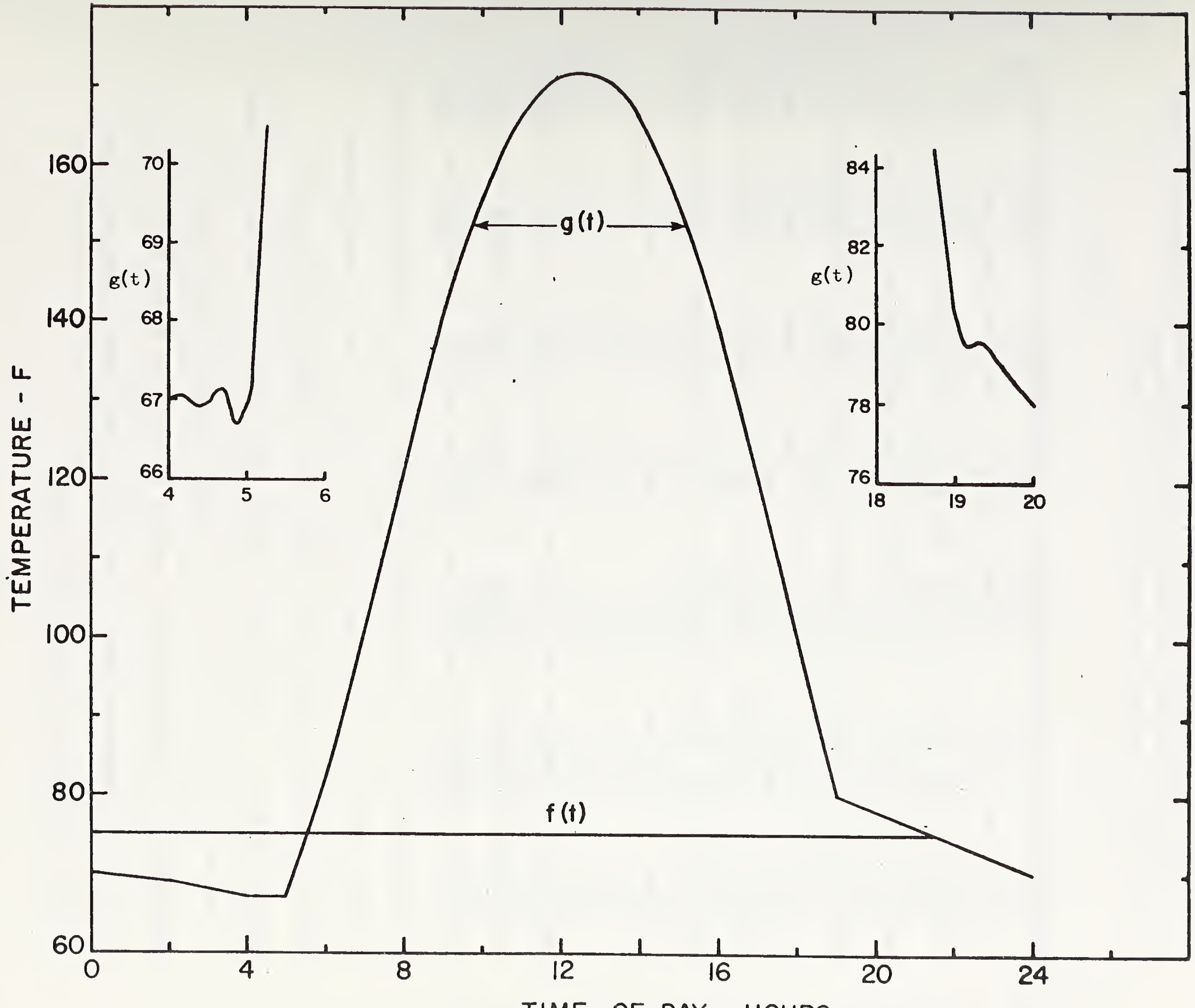

Figure 1. Temperature variations for g(t) and f(t) used in verification of response factors.

Considering only the steady periodic condition, the residues at the poles at $p=0$ and $p=\pm iw_n$ can be found for the surface temperatures. Heat flow at the surfaces can then be obtained from (4) and (5).

The six examples of Table 2 were used in determination of heat flux for both the response factor and analytical solution. The temperature variation, g(t) shown in Figure 1 was used for the test cases, with f(t)=0 and R_1 = 0.68 and R_2 = .333 hft^{2}F/Btu. The coefficients C_o, C_n and D_n were determined from 96 points where there was linear interpolation between the hourly points. This was necessary to assure a nearly linear variation in temperature between the hourly points for the function g(t). When coefficents were determined for 24 and 48 points the agreement between the heat flux for the analytical and response factor was not as good.

Heat flux was computed both from the response factors [(23) and (24)] and the conduction transfer function (26). For all cases examined the heat flux at the inside surface was less than one percent different from the values computed for the analytical solution. At the outside surface, the agreement was not as good, particularly around 5 and 19 hours, where there are sudden changes in the slope of the temperature curve (as shown on Figure 1). The detail figures show that the function g(t) used in the analytical solution varies considerably from the linear form required by the response-factor method in the time period 4 to 5 and 19 to 20. It is expected that if more points were used, there would be a better agreement at the outside surface.

Table 5. Comparison of Heat Flux Values Calculated from Response-Factor and Analytical Methods Btu/h ft^2

	Case 2.				Case 3.			
	Outside Surface		Inside Surface		Outside Surface		Inside Surface	
	Resp.	Anal.	Resp.	Anal.	Resp.	Anal.	Resp.	Anal.
1	1.128	1.103	.358	.358	39.450	39.437	-5.468	-5.468
2	.487	.503	.442	.442	34.607	34.609	-4.867	-4.866
3	1.177	1.152	.479	.478	32.679	32.668	-4.295	-4.295
4	1.268	1.195	.542	.542	30.798	30.768	-3.763	-3.763
5	.652	.304	.604	.604	27.171	27.036	-3.271	-3.270
6	- 9.247	- 9.338	.430	.434	- 2.856	- 2.920	-2.816	-2.816
7	-15.479	-15.366	- .489	- .486	- 38.739	- 38.726	-2.408	-2.408
8	-16.531	-16.412	-1.933	-1.933	- 65.508	- 65.481	-2.110	-2.109
9	-16.683	-16.552	-3.456	-3.456	- 84.931	- 84.961	-2.012	-2.011
10	-15.951	-15.807	-4.861	-4.862	- 96.946	- 96.894	-2.170	-2.169
11	-14.261	-14.108	-6.047	-6.048	-101.138	-101.076	-2.590	-2.589
12	-11.537	-11.379	-6.930	-6.932	- 96.975	- 96.904	-3.238	-3.237
13	- 7.701	- 7.565	-7.432	-7.434	- 83.896	- 83.828	-4.058	-4.057
14	- 3.379	- 3.286	-7.488	-7.490	- 63.289	- 63.233	-4.978	-4.978
15	.709	.783	-7.091	-7.093	- 37.984	- 37.933	-5.916	-5.916
16	5.175	5.183	-6.284	-6.284	- 8.151	- 8.124	-6.786	-6.787
17	8.621	8.604	-5.110	-5.111	21.992	22.007	-7.509	-7.510
18	12.274	12.076	-3.661	-3.662	53.149	53.093	-8.015	-8.016
19	11.223	10.820	-2.065	-2.064	71.912	71.763	-8.254	-8.255
20	1.296	1.332	- .811	- .806	58.711	58.703	-8.195	-8.197
21	1.226	1.242	- .333	- .332	52.360	52.354	-7.857	-7.858
22	1.366	1.371	- .113	- .113	48.294	48.290	-7.332	-7.332
23	1.520	1.517	.055	.055	45.399	45.394	-6.722	-6.722
24	1.676	1.648	.214	.214	43.178	43.164	-6.901	-6.091

Comparison of calculated heat flux values from the analytical and response-factor methods is shown in Table 5 for Cases 2 and 3.

5. CONCLUSIONS

Formulas for calculating thermal response factors for plane multi-layer constructions have been developed as given by equations (19), (20), and (21). A computer program to obtain response factors based on these formulas is found in the Appendix. A comparison was made of the time for computation of response factors between the matrix algebra method of Kusuda [1] and the method given in this paper. A considerable saving in computation time is realized by the methods of this paper.

Presently, the response-factor calculations for constructions with air spaces assume only a purely thermal resistance effect of air spaces during dynamic temperature variations [2]. The heat transfer across an air space involves the nature of the bounding surfaces, the intervening air, orientation of the space and the direction of heat flow: and hence the three modes of heat transfer--radiation, convection, and conduction--influence heat flow in an air space. It would be impractical to simulate the three modes of heat transfer for constantly changing air space surface temperatures, but it is felt that a reasonable approximation should include both the heat capacity and air space thickness for dynamic heat transfer calculations. These were included in the computer program of the Appendix. The introduction of these changes gives different values for the response factors and resulting heat-flux amplitudes.

Response factors are analytical formulations from the partial differential equations for heat conduction. When properly applied, the response-factor method gives correct values for temperature and heat flow for conduction heat transfer problems.

6. REFERENCES

1. T. Kusuda, Thermal Response Factors for Multi-Layer Structures of Various Conduction Systems, ASHRAE Transactions, Vol. 75, 1969.

2. T. Kusuda, NBSLD, The Computer Program for Heating and Cooling Loads in Buildings, NBS-BSS-69, U.S. Government Printing Office, Wash., D.C. 20402.

7. CONVERSION FACTORS TO METRIC (S.I.) UNITS

Physical Quantity	To Convert From	To	Multiply By
Length	ft	m	3.0480 E-1
Area	ft^2	m^2	9.2903 E-2
Temperature	F	C	(F-32)/1.8
Density	lbm/ft^3	kg/m^3	1.6018 E+1
Thermal Conductivity	Btu/h ftF	W/mK	1.7296 E+0
Thermal Resistance	h ft^2F/Btu	m^2K/W	1.7623 E-1
Thermal Diffusivity	ft^2/h	m^2/s	2.9900 E-3
Heat Flux	Btu/h ft^2	W/m^2	3.1525 E+0
Specific Heat	Btu/lbmF	j/kgK	4.1840 E+3
Time	h	s	3.6000 E+3

APPENDIX A

Appendix A gives a Fortran listing of computations for the response factors and includes input of data identical to that of Kusuda [1].

```
C      PROGRAM FOR COMPUTING RESPONSE FACTORS OF MULTI-LAYER BUILDING
C       CONSTRUCTIONS (ONE TO SEVEN LAYERS) FOR THERMAL CONDUCTION
C       APPLICATIONS
C         INPUT
C     DEL= TIME INCREMENT  (EQU 15)
C     JA= NUMBER OF LAYERS IN CONSTRUCTION (MAY INCLUDE SURFACE FILMS)
C   THERMOPHYSICAL PROPERTIES OF EACH LAYER (ONE CARD PER LAYER-10F7.0)
C    SURFACE FILM RESISTANCES MAY BE INCLUDED AS LAYERS- IN 'RE' ONLY
C     IF INCLUDED FOR BOTH SIDES - 'JA' SHOULD NOT EXCEED 9
C    ORDERING OF LAYERS FROM INSIDE TO OUTSIDE
C     L = THICKNESS OF EACH LAYER
C     K = THERMAL CONDUCTIVITY OF LAYER
C     D = DENSITY OF LAYER
C     SP= SPECIFIC HEAT OF LAYER
C     RE= THERMAL RESISTANCE (FOR AIR SPACES OR SURFACE FILMS ONLY)
C   DESCRIPTION OF EACH LAYER  (ONE CARD PER LAYER-6A6)
C     RM= CONTAINS DESCRIPTIONS FOR EACH LAYER (HOLLERITH)
C   BLANK CARD INDICATES NO MORE CONSTRUCTIONS TO BE INPUTTED
C
C
C       SUBROUTINE ROOTS - COMPUTES ROOTS OF EQU 10
C
C       SUBROUTINE ABC - COMPUTES (I=1) TERMS FOR EQUS 16,17,18
C                                 (I=0) SUMMATION TERMS FOR EQUS 10,11
C
C
C         OUTPUT
C     THERMOPHYSICAL PROPERTIES OF EACH LAYER AND DESCRIPTIONS
C    L(N),K(N),D(N),SP(N),RE(N),RM(I,N),I=1,6 (EXCLUDING SURFACE FILMS)
C     STATEMENT - SURFACE FILMS INCLUDED OR EXCLUDED
C     X = PRINTOUT OF TERMS USED IN COMPUTING EQUS 16,17,18
C     BB,BC,BD,BE,BF,BG - TERMS OF EQUS 16,17,18
C     Y = ROOTS OF EQU 10
C     CR= COMMON RATIO EQU 22
C    RESPONSE FACTORS
C     XX(N),YY(N),ZZ(N), N=1,IB  IB DEFINED BY EQU 22
C
      PARAMETER F=60,G=100
      DIMENSION S(64),B(7),C(6),E(6),L(8),K(8),D(8),SP(8),RE(8),RM(8,6),
     1AL(7),X(F),Y(F),W(F),T(F),XX(G),YY(G),ZZ(G)
      COMMON/CBA/S,C
      REAL K,L
  100 FORMAT (10I7)
  102 FORMAT (10F7.0)
  105 FORMAT (6A6)
  201 FORMAT (10F12.6)
  202 FORMAT (3F16.8)
  203 FORMAT (I10,F11.4,F10.3,F10.1,F10.3,F8.2,2X,6A6)
  205 FORMAT (26H BUILDING CONSTRUCTION NO.I4)
  206 FORMAT (33H ROOTS OF CHARACTERISTIC EQUATION)
  207 FORMAT (24H RESPONSE FACTORS  X,Y,Z)
  208 FORMAT(1H1)
  209 FORMAT(1H )
  210 FORMAT (56H RESPONSE FACTORS ARE CALCULATED FROM SURFACE TO SURFAC
     AE)
  211 FORMAT (56H RESPONSE FACTORS INCLUDE SURFACE FILM RESISTANCES - R1
     A=F6.3,8H AND R2=F6.3)
  212 FORMAT (14H COMMON RATIO=F9.6)
      READ (5,102) DEL
      IR=0
```

```
 97 WRITE (6,208)
 99 READ (5,100) JA
    J=JA
    R1=.0
    R2=.0
    IF (J.EQ.0) GO TO 200
    READ (5,102) L(1),K(1),D(1),SP(1),RE(1)
    IA=2
    IF (D(1).GT..001) GO TO 98
    R1=RE(1)
    J=J-1
    IA=1
 98 DO 101 N=IA,J
101   READ (5,102) L(N),K(N),D(N),SP(N),RE(N)
    IR=IR+1
    READ (5,105) (RM(1,I),I=1,6)
103 DO 104 N=IA,J
104   READ (5,105) (RM(N,I),I=1,6)
    IF (D(J).GT. .001) GO TO 106
    R2=RE(J)
    J=J-1
106 IF (J.GT.7.OR.J.EQ.0) GO TO 200
    DO 110 N=1,J
      IF (D(N)) 109,107,109
107   AL(N)=.75
      IF (L(N).LT..001) L(N)=.083333
      K(N)=L(N)/RE(N)
      GO TO 110
109   AL(N)=K(N)/(D(N)*SP(N))
110   CONTINUE
    AA=.0
 19 DO 20 N=1,7
      B(N)=.0
      IF (N.LE.J) AA=AA+L(N)/K(N)
 20   IF (N.LE.J) B(N)=L(N)/SQRT(AL(N))
    AD=1.
    IF (J.EQ.1) GO TO 1
    I=J-1
    DO 21 N=1,I
      E(N)=K(N+1)*SQRT(AL(N)/AL(N+1))/K(N)
      C(N)=(1.-E(N))/(1.+E(N))
 21   AD=2.*AD/(1.+E(N))
    GO TO (1,2,3,4,5,6,7),J
  7 S(32)=B(7)-B(6)+B(5)-B(4)+B(3)-B(2)+B(1)
    S(31)=B(7)-B(6)+B(5)-B(4)+B(3)+B(2)+B(1)
    S(30)=B(7)-B(6)+B(5)-B(4)-B(3)+B(2)+B(1)
    S(29)=B(7)-B(6)+B(5)+B(4)-B(3)+B(2)+B(1)
    S(28)=B(7)-B(6)-B(5)+B(4)-B(3)+B(2)+B(1)
    S(27)=B(7)-B(6)+B(5)-B(4)-B(3)-B(2)+B(1)
    S(26)=B(7)-B(6)+B(5)+B(4)-B(3)-B(2)+B(1)
    S(25)=B(7)-B(6)-B(5)+B(4)-B(3)-B(2)+B(1)
    S(24)=B(7)-B(6)+B(5)+B(4)+B(3)-B(2)+B(1)
    S(23)=B(7)-B(6)-B(5)+B(4)+B(3)-B(2)+B(1)
    S(22)=B(7)-B(6)-B(5)-B(4)+B(3)-B(2)+B(1)
    S(21)=B(7)-B(6)-B(5)-B(4)-B(3)-B(2)+B(1)
    S(20)=B(7)-B(6)-B(5)-B(4)-B(3)+B(2)+B(1)
    S(19)=B(7)-B(6)-B(5)-B(4)+B(3)+B(2)+B(1)
    S(18)=B(7)-B(6)-B(5)+B(4)+B(3)+B(2)+B(1)
    S(17)=B(7)-B(6)+B(5)+B(4)+B(3)+B(2)+B(1)
  6 S(16)=B(7)+B(6)-B(5)+B(4)-B(3)+B(2)+B(1)
```

```
      S(15)=B(7)+B(6)-B(5)+B(4)-B(3)-B(2)+B(1)
      S(14)=B(7)+B(6)-B(5)+B(4)+B(3)-B(2)+B(1)
      S(13)=B(7)+B(6)-B(5)-B(4)+B(3)-B(2)+B(1)
      S(12)=B(7)+B(6)-B(5)-B(4)-B(3)-B(2)+B(1)
      S(11)=B(7)+B(6)-B(5)-B(4)-B(3)+B(2)+B(1)
      S(10)=B(7)+B(6)-B(5)-B(4)+B(3)+B(2)+B(1)
      S( 9)=B(7)+B(6)-B(5)+B(4)+B(3)+B(2)+B(1)
    5 S( 8)=B(7)+B(6)+B(5)-B(4)+B(3)-B(2)+B(1)
      S( 7)=B(7)+B(6)+B(5)-B(4)+B(3)+B(2)+B(1)
      S( 6)=B(7)+B(6)+B(5)-B(4)-B(3)+B(2)+B(1)
      S( 5)=B(7)+B(6)+B(5)-B(4)-B(3)-B(2)+B(1)
    4 S( 4)=B(7)+B(6)+B(5)+B(4)-B(3)+B(2)+B(1)
      S( 3)=B(7)+B(6)+B(5)+B(4)-B(3)-B(2)+B(1)
    3 S( 2)=B(7)+B(6)+B(5)+B(4)+B(3)-B(2)+B(1)
    2 S( 1)=B(7)+B(6)+B(5)+B(4)+B(3)+B(2)+B(1)
      GØ TØ 9
    1 S(1)=B(1)
    9 DØ 8 N=1,32
        I=N+32
    8   S(I)=S(N)-2.*B(1)
      AB=E(1)/SQRT(AL(1))
      AC=E(J)/SQRT(AL(J))
      V1=R1*AB
      V2=R2*AC
      I=1
      CALL ABC(CA,X,J,I)
      CA=V1*V2
      CB=V2+V1
      CC=V2-V1
      BA=X(2)+X(6)+CB*X(1)+CC*X(5)
      BB=AB*(X(1)-X(5))/BA
      BC=AC*(X(1)+X(5))/BA
      BD=AC*AD/BA
      BH=(X(4)+X(8))/6.+CA*(X(2)-X(6))+(CB*X(3)+CC*X(7))/2.
      BI=(X(3)-X(7))/2.+V2*(X(2)-X(6))
      BJ=(X(3)+X(7))/2.+V1*(X(2)-X(6))
      BE=AB*(BA*BI-BH*(X(1)-X(5)))/(DEL*BA*BA)
      BF=AC*(BA*BJ-BH*(X(1)+X(5)))/(DEL*BA*BA)
      BG=-AC*AD*BH/ (DEL*BA*BA)
      X(1)=SQRT(BA/BH)
      CALL RØØTS(AB,AC,AD,V1,V2,DEL,Y,X,T,W,J,M)
      XX(1)=BB+BE
      ZZ(1)=BC+BF
      YY(1)=BD+BG
      XX(2)=-BE
      ZZ(2)=-BF
      YY(2)=-BG
      DØ 50 N=1,M
        XX(1)=XX(1)-X(N)
        ZZ(1)=ZZ(1)-W(N)
        YY(1)=YY(1)-T(N)
        CA=EXP(-DEL*Y(N)**2)-2.
        XX(2)=XX(2)-X(N)*CA
        ZZ(2)=ZZ(2)-W(N)*CA
   50   YY(2)=YY(2)-T(N)*CA
      CR=EXP(-DEL*Y(1)**2)
      DØ 56 I=3,G
        XX(I)=.0
        YY(I)=.0
        ZZ(I)=.0
```

```
      IB=1
      CC=I-3
      CA=CC*Y(1)**2*DEL
      IF (CA.GT.20.) GO TO 59
     DO 55 N=1,M
      CA=DEL*Y(N)**2
      CB=CC*CA
      IF (CB.GT.40.) GO TO 56
      CD=EXP(-CA)
      CD=EXP(-CB)*(1.-CD)**2
      XX(I)=XX(I)-X(N)*CD
      ZZ(I)=ZZ(I)-W(N)*CD
  55  YY(I)=YY(I)-T(N)*CD
  52  CA=ABS(XX(I)/XX(I-1)-CR)+ABS(YY(I)/YY(I-1)-CR)
      CA=CA+ABS(ZZ(I)/ZZ(I-1)-CR)
  56  IF (CA.LT..00003) GO TO 59
  59 WRITE (6,205) IB
     DO 204 I=1,J
 204  WRITE (6,203) I,L(I),K(I),D(I),SP(I),RE(I),(RM(I,N),N=1,6)
     WRITE (6,209)
     IF (J.EQ.JA) WRITE (6,210)
     IF (J.NE.JA) WRITE (6,211) R1,R2
     WRITE (6,206)
     WRITE (6,201) (Y(N),N=1,5)
     WRITE (6,209)
     WRITE (6,207)
     WRITE (6,202) (XX(N),YY(N),ZZ(N),N=1,IB)
     WRITE (6,212) CR
     GO TO 97
 200 STOP
     END



     SUBROUTINE ABC(X,Z,J,I)
     DIMENSION Z(1),T(64),V(64,4)
     COMMON/CBA/S(64),C(6)
     K=1
     IF (I.EQ.1) GO TO 20
     M=1
     IF (J.GT.1) M=2**J/4
     DO 8 N=1,M
     DO 8 I=1,2
      L=N
      IF (I.EQ.2) L=N+32
      B=X*S(L)
      A=SIN(B)
      B=COS(B)
      V(L,1)=A
      V(L,2)=B
      V(L,3)=B*S(L)
   8  V(L,4)=A*S(L)
  12 DO 14 N=1,64
  14  T(N)=V(N,K)
  15 Y=.0
     W=.0
  10 GO TO (1,2,3,4,5,6,7),J
   7 A=C(1)*(C(2)*T(24)+C(3)*T(26)+C(4)*(T(27)+C(2)*C(3)*T(32)))+T(17)
     A=C(5)*(A+C(2)*(C(3)*T(29)+C(4)*T(30))+C(3)*C(4)*T(31))+C(2)*T(20)
```

```
      B=C(4)*(T(18)+C(2)*(C(1)*T(23)+C(3)*T(28))+C(1)*C(3)*T(25))
      A=A+B+C(3)*(T(19)+C(1)*C(2)*T(22))+C(1)*T(21)
      Y=Y+C(6)*A
      A=C(2)*(C(3)*C(4)*T(60)+C(3)*C(5)*T(61)+C(4)*C(5)*T(62))
      B=C(2)*T(52)+C(3)*T(51)+C(4)*T(50)+C(5)*(T(49)+C(3)*C(4)*T(63))
      B=T(53)+C(1)*(A+B)+C(3)*(C(4)*T(57)+C(5)*T(58))+C(4)*C(5)*T(59)
      A=C(2)*(C(3)*T(54)+C(4)*T(55)+C(5)*(T(56)+C(3)*C(4)*T(64)))
      W=W+C(6)*(A+B)
    6 A=C(4)*(T(9)+C(1)*(C(2)*T(14)+C(3)*T(15)))+C(2)*T(11)+C(1)*T(12)
      Y=Y+C(5)*(A+C(3)*(T(10)+C(2)*(C(1)*T(13)+C(4)*T(16))))
      A=T(44)+C(1)*(C(4)*T(41)+C(3)*T(42)+C(2)*T(43))+C(3)*C(4)*T(47)
      W=W+C(5)*(A+C(2)*(C(3)*T(45)+C(4)*(T(46)+C(1)*C(3)*T(48))))
    5 Y=Y+C(4)*(C(1)*T(5)+C(2)*T(6)+C(3)*(T(7)+C(1)*C(2)*T(8)))
      W=W+C(4)*(T(37)+C(1)*(C(2)*T(38)+C(3)*T(39))+C(2)*C(3)*T(40))
    4 Y=Y+C(3)*(C(1)*T(3)+C(2)*T(4))
      W=W+C(3)*(T(35)+C(1)*C(2)*T(36))
    3 Y=Y+C(1)*C(2)*T(2)
      W=W+C(2)*T(34)
    2 Y=Y+T(1)
      W=W+C(1)*T(33)
      Z(K)=Y
      Z(K+4)=W
   16 K=K+1
      IF (I.EQ.1) GO TO 22
      IF (K.LE.4) GO TO 12
      RETURN
    1 Z(K)=T(1)
      Z(K+4)=.0
      GO TO 16
   20 DO 21 N=1,64
   21   T(N)=1.
      GO TO 15
   22 DO 23 N=1,64
   23   T(N)=T(N)*S(N)
      IF (K.LE.4) GO TO 15
      RETURN
      END


      SUBROUTINE ROOTS(AB,AC,AD,V1,V2,DEL,B,Z,P,R,J,M)
      DIMENSION B(1),Z(1),P(1),R(1),Y(8)
      I=0
      V=V1*V2
      G=V2+V1
      H=V2-V1
      F=2.
      U=.02
      M=1
      K=0
      A=.005
      CALL ABC(A,Y,J,I)
      D=Y(1)+Y(5)+A*(G*Y(2)+H*Y(6)-A*V*(Y(1)-Y(5)))
      X=Z(1)
      E=.15*X
    4 CALL ABC(X,Y,J,I)
      A=Y(1)+Y(5)+X*(G*Y(2)+H*Y(6)-X*V*(Y(1)-Y(5)))
      T=(1.-V*X*X)*Y(3)+(1.+V*X*X)*Y(7)-2.*X*V*(Y(1)-Y(5))
      IF (ABS(A).LT.1.E-6) GO TO 7
```

```
      IF (K.GT.2) GO TO 6
C     INCREMENTING TO OBTAIN FIRST ZERO - AND APPROXIMATE FOR OTHERS
      IF (A*D) 5,5,8
    5 X=X-E
      E=E/2.
      IF (M.EQ.1) GO TO 4
      K=K+1
      GO TO 4
    8 X=X+E
      GO TO 4
C     NEWTON-RAPHSON METHOD FOR SOLVING EQU 10
    6 Q=T+G*(Y(2)-X*Y(4))+B*(Y(6)-X*Y(8))
      X=X-A/Q
      K=K+1
C     NOT TO EXCEED 8-TH ITERATION
      IF (K.GT.8) GO TO 7
      GO TO 4
    7 B(M)=X
      A=EXP(-X*X*DEL)
      Q=T+G*(Y(2)-X*Y(4))+B*(Y(6)-X*Y(8))
      E=2.*A/(X*X*DEL*Q)
      Z(M)=E*(Y(2)-Y(6)-X*V2*(Y(1)-Y(5)))*AB
      P(M)=E*AC*AD
      R(M)=E*AC*(Y(2)+Y(6)-X*V1*(Y(1)-Y(5)))
      IF (X*X*DEL.GT.30.) RETURN
      IF (M.EQ.60) RETURN
      K=0
      IF (M.GT.1) GO TO 1
      IF (X.GT..5) GO TO 1
      F=3.5
      U=.002
    1 X=X+U
      CALL ABC(X,Y,J,I)
      D=Y(1)+Y(5)+X*(G*Y(2)+B*Y(6)-X*V*(Y(1)-Y(5)))
      IF (M.EQ.1) E=X/4.
      IF (M.GT.1) E=(X-B(M-1))/F
      M=M+1
      X=X+E
      GO TO 4
      END
```

FEDERAL INFORMATION PROCESSING STANDARD SOFTWARE SUMMARY

01. Summary date (Yr. Mo. Day): 77 12 21	02. Summary prepared by (*Name and Phone*): Bradley A. Peavy (301) 921-3503	03. Summary action: New ☒ Replacement ☐ Deletion ☐
04. Software date (Yr. Mo. Day):	05. Software title: Determination of Thermal Response Factors	Previous Internal Software ID
06. Short title		07. Internal Software ID

08. Software type
- ☐ Automated Data System
- ☒ Computer Program
- ☐ Subroutine/Module

09. Processing mode
- ☐ Interactive
- ☒ Batch
- ☐ Combination

10. Application area

General:
- ☐ Computer Systems Support/Utility
- ☒ Scientific/Engineering
- ☐ Bibliographic/Textual
- ☐ Management/Business
- ☐ Process Control
- ☐ Other

Specific:

11. Submitting organization and address

National Bureau of Standards
Washington, D.C. 20234

12. Technical contact(s) and phone

B.A. Peavy
Building 226, Room B104
(301) 921-3503

13. Narrative

Program computes thermal response factors of multi-layer building constructions (one to seven layers) for thermal conduction applications. Provision is made for including or excluding surface film thermal resistances.Input includes time increment (1 hour, 1/2 hour, etc.), number of layers, thermal properties and descriptions of each layer. Output includes the thermal response factors, X, Y and Z. Main program is amendable to use as a subprogram in a larger program for use in thermal conduction applications.

14. Keywords

Dynamic conduction heat transfer, thermal response factors

15. Computer manuf'r and model	16. Computer operating system	17. Programing language(s)	18. Number of source program statements
UNIVAC 1108		FORTRAN V	
19. Computer memory requirements	**20. Tape drives**	**21. Disk/Drum units**	**22. Terminals**
	None	None	

23. Other operational requirements

24. Software availability

Available ☒ Limited ☐ In-house only ☐

25. Documentation availability

Available ☐ Inadequate ☐ In-house only ☐

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15. SUPPLEMENTARY NOTES

16. ABSTRACT *(A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)*

New formulas for calculating thermal response factors for multiple-layer construction have been developed by a rigorous derivation. A comparison was made of the time for computation between the presently used matrix algebra method and the method given in this paper. Results were obtained using the new method in one-fiftieth to one-half of the computational time necessary to obtain solutions from the matrix algebra method.

Comparisons with another analytical method were performed to verify the accuracy of the response-factor technique.

17. KEY WORDS *(six to twelve entries; alphabetical order; capitalize only the first letter of the first key word unless a proper name; separated by semicolons)* Dynamic conduction heat transfer; heat transfer; thermal response factor; verification.

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